Electromagnetic fields in heavy-ion collisions

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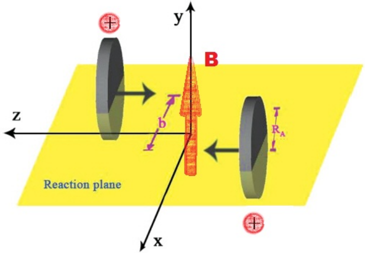
Contents

Introduction and motivations

Magnetic field in HIC (I)

- Heavy-Ion Collisions generate: many particles, deconfined matter, ..., and strong magnetic field.
- Imagine noncentral collision $\Rightarrow$ Large B field in y direction. No (or small) E field.



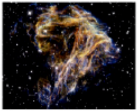
- How strong? A crude estimate:
  - RHIC Au+Au collision, $Z = 79$, $\sqrt{s} = 200$ GeV ($\Rightarrow v_z \simeq 0.99995c$), impact parameter $b = 5$ fm
  - The B field at the colliding time, $t = 0$. Biot-Savart law

$$eB_y \sim 2 \times \gamma \frac{e^2}{4\pi} Z v_z (2/b)^2 \approx 40 m_\pi^2 \sim 10^{19} \mathrm{Gauss}$$

Magnetic field in HIC (II)

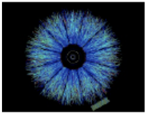
- How strong? Comparison **From D. Kharzeev**

Comparison of magnetic fields



The Earths magnetic field	0.6 Gauss
A common, hand-held magnet	100 Gauss
The strongest steady magnetic fields achieved so far in the laboratory	4.5×10^{5} Gauss
The strongest man-made fields ever achieved, if only briefly	10^{7} Gauss
Typical surface, polar magnetic fields of radio pulsars	10^{13} Gauss
Surface field of Magnetars	10^{15} Gauss

http://solomon.as.utexas.edu/~duncan/magnetar.html



Heavy ion collisions: the strongest magnetic field ever achieved in the laboratory

Off central Gold-Gold Collisions at 100 GeV per nucleon

$eB(\tau=0) \sim 10^{19}$ Gauss

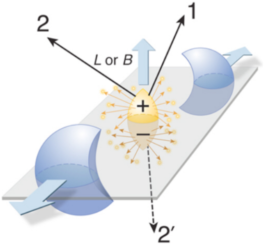
Chiral magnetic effect

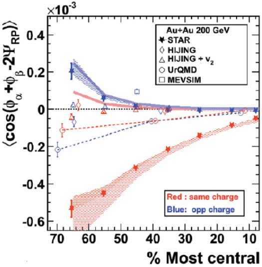
- Such a strong B field may influence the dynamics of QGP
- Topological charge + magnetic field = chiral magnetic effect (CME)

  Kharzeev 2004, Kharzeev, Mclerran, and Warringa, Fukushima 2008:

$$\mathbf{J}_V = \frac{N_c e}{2\pi^2}\mu_A \mathbf{B}$$

- Phenomenology: charge-charge azimuthal correlation. **STAR 2009-2012, ALICE 2012**





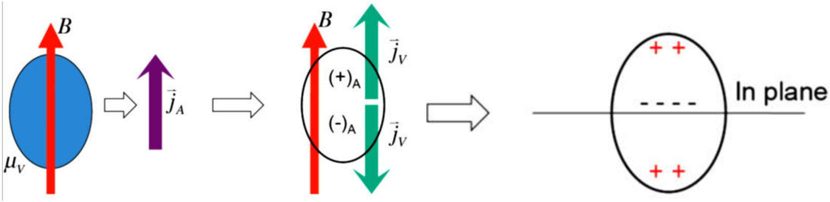
- Signal for local parity violation of QCD?! Need more theoretical and experimental explorations. **Wang 2010, Pratt 2010, Liao, Bzdak, and Koch 2010, 2011, 2012...**

Chiral magnetic wave (I)

- A dual effect to chiral magnetic effect: chiral separation effect (CSE)

$$\mathbf{J}_V = \frac{N_c e}{2\pi^2}\mu_A \mathbf{B} \;\Rightarrow\; \mathbf{J}_A = \frac{N_c e}{2\pi^2}\mu_V \mathbf{B}$$

- HIC contain net μ_V $\rightarrow$ CSE $\rightarrow$ chirality separation $\rightarrow$ CME $\rightarrow$ charge separation $\rightarrow$ CSE $\rightarrow$ $\cdots \Rightarrow$ Chiral magnetic wave (CMW)
- CMW transports charge and chirality $\Rightarrow$Electric quadrupole of QGP.**Burnier, Kharzeev, Liao, and Yee 2011**



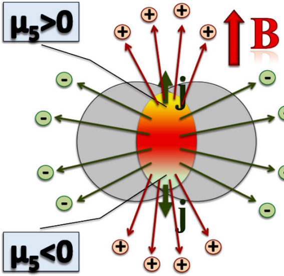
Picture from G. Wang

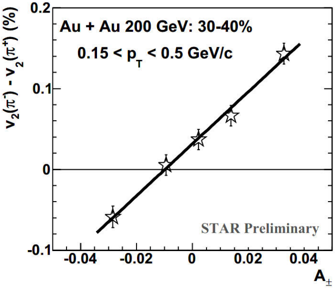
Chiral magnetic wave (II)

- Electric quadrupole $\Rightarrow$ more π^+ fly up and down, more π^- fly in-plane $\Rightarrow$ Anisotropic emission of charged pion: Elliptic flow v_2.

$$\frac{dN_{\pi^\pm}(\phi)}{d\phi} \sim 1 + 2v_2(\pi^\pm)\cos 2(\phi - \Psi_{\mathrm{RP}}) + \cdots$$

- CMW $\Rightarrow$ $v_2(\pi^-) > v_2(\pi^+)$: $v_2(\pi^-) - v_2(\pi^+) \approx r A_\pm$: linear approx. in net charge asymmetry $A_\pm = (N_+ - N_-)/(N_+ + N_-)$.

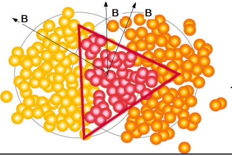


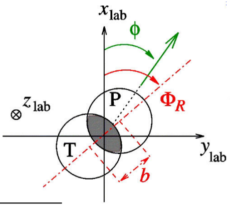


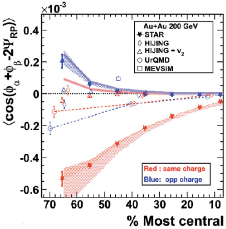
- STAR measurement (2012) qualitatively coincides with CMW prediction!

Pay more attention on B!!

- These "anomalous" phenomena are interesting and important: Evidence of parity violation in QCD? Evidence of chiral symmetry restoration? $\cdots$.
- Magnetic field plays a key role.
  - More careful computation of $\mathbf{B}$[1].
    All observables sensitive to the magnitude of $\mathbf{B}$.
  - How $\mathbf{B}$ varies from event to event[2].
    Experiment counts many events, all observables fluctuate from e to e.
  - How $\mathbf{B}$ correlated to the matter geometry[3].
    Correlations, elliptic flows are measured w.r.t. the matter geometry.



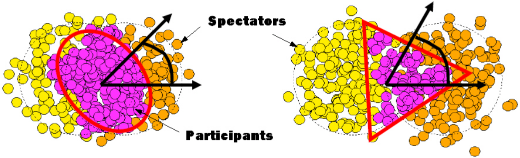




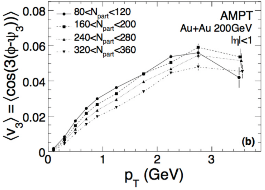
[1] **Skokov et al 2009, Voronyuk et al 2011, Bzdak and Skokov 2011, Deng and Huang 2012.**
[2] **Bzdak and Skokov 2011, Deng and Huang 2012.**
[3] **Bloczynski, Huang, Zhang, and Liao 2012.**

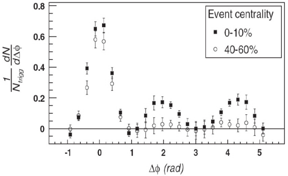
Event-by-event fluctuation in HIC

- Nucleon distribution. In average: Woods-Saxon. Varies from one nucleus to another ⇒ real collision geometry also varies



- This event-by-event fluctuation may be responsible to many observables: odd-harmonic flow $v_3, v_5, \ldots$, away-side double peak, etc





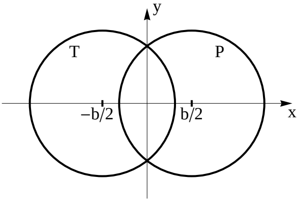
- **On event-by-event basis, we calculate the magnetic field and study its azimuthal correlation to the matter geometry.**

Numerical Simulations[4]

[4] Deng and Huang PRC85(2012)044907, Bloczynski, Huang, Zhang, and Liao arXiv:1209.6594

Setup

- Monte Carlo simulates nucleon distributions in nuclei for each event.



- After collision, the parton and nuclear remnant distributions are simulated by HIJING (**Wang and Gylassy 1991**).
- Apply Lienard-Wiechert potentials to each event

$$e\mathbf{E}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{R}_n - R_n\mathbf{v}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2),$$

$$e\mathbf{B}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2),$$

where $\mathbf{R}_n$ is the relative position of the field point to the source point and$\mathbf{v}_n$ at the retarded time $t_n = t - |\mathbf{R}_n|$.

Setup

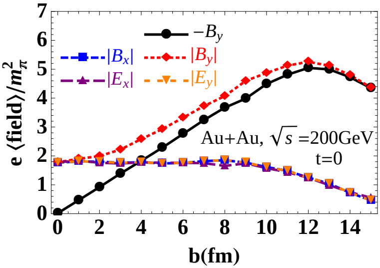
- Singularities at short distance
  - Diverge $\sim 1/r^2$
  - Run large enough numbers of events, $\lim_{r_\Lambda \to 0} \lim_{N\to\infty} \sum_{r_i > r_\Lambda} 1/r_i^2 \sim \int d^3r/r^2$, finite
- Lienard-Wiechert is classical. However, $eB \gg m_e^2$, QED effect?
  - When $eB \gg m_e^2$, QED effective lagrangian **Heisenberg and Euler 1936**

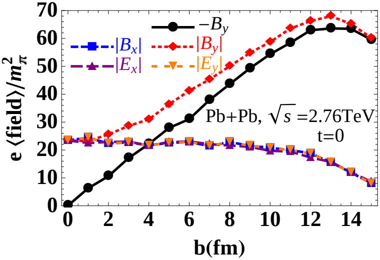
$$\mathcal{L}_{\rm eff} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu}\left[1 - \frac{e^2}{24\pi^2}\ln\frac{e^2|F|^2}{m_e^4}\right] + eA_\mu j^\mu$$

  - The EOM is still Maxwell-type but with a renormalized $e^2 \to \tilde{e}^2 = e^2/\left[1 - \frac{e^2}{24\pi^2}\ln\frac{e^2|F|^2}{m_e^4}\right]$
  - Even for $eB \sim 100 m_\pi^2$, results change only a few percent.
- Lienard-Wiechert potentials can work quite well.

Impact parameter dependence

- EM field at $t = 0$ and $\mathbf{r} = 0$, the center of the collision region

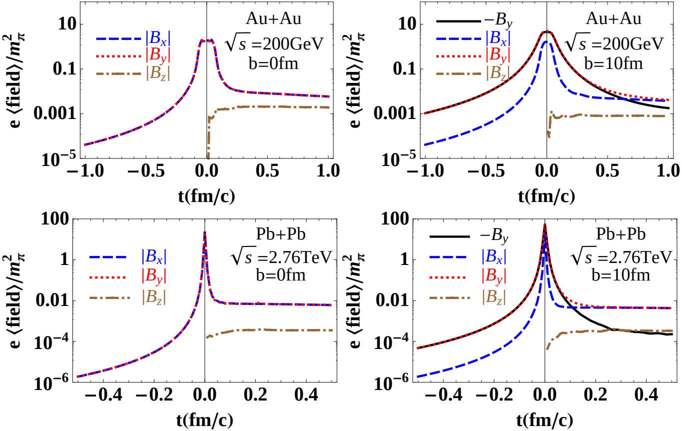




- Event-by-event fluctuation causes strong B and E fields even for $b = 0$. Several m_π^2 for RHIC. Several tens of m_π^2 for LHC.
- Fluctuation-caused fields is not sensitive to b, event-averaged B_y is linear in b when b not large
- We also examine: $e \cdot \mathrm{Field} \propto \sqrt{s}$
- Fluctuation does not generate large longitudinal fields B_z, E_z.

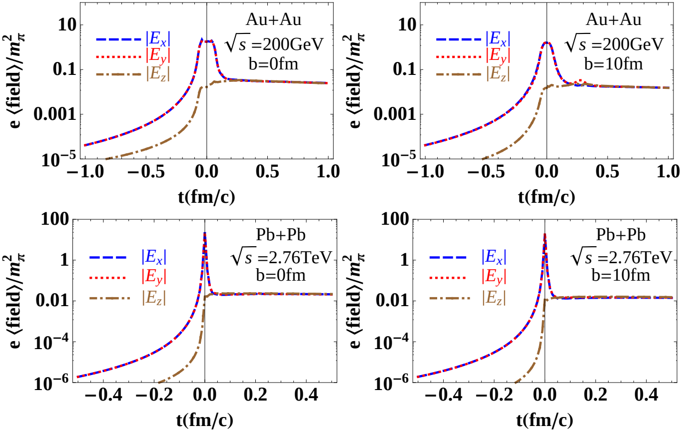
Time evolution of B field

- Spectators fly away, fields decay in a very short timescale $\tau \sim 2R_A/\gamma \sim 4R_A m_N/\sqrt{s}$
- Afterwards, remnant nucleons dominate, fields decay slowly
- z-components always much smaller than x, y-components



Time evolution of E field

- Fluctuation-caused E fields have similar time evolving behavior as B fields
- After a short time scale $\sim 4R_A m_N/\sqrt{s}$, E_z becomes comparable with $E_{x,y}$



Electromagnetic response of the QGP (I)

- If QGP is formed, how EM fields evolve?
- Consider the Maxwell equations + Ohm's law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \;\; \nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}$$
$$\nabla \cdot \mathbf{B} = 0, \;\; \nabla \cdot \mathbf{E} = \rho = 0$$
$$\mathbf{J} = \sigma \left(\mathbf{E} + \mathbf{v} \times \mathbf{B} \right), \;\; \sigma = \text{electric conductivity}$$

- One can derive the induction equations.
  Blue: convection terms. Green: diffusion terms

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{\sigma}\left(\nabla^2 \mathbf{B} - \frac{\partial^2 \mathbf{B}}{\partial t^2} \right),$$
$$\frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{v}}{\partial t} \times \mathbf{B} = \mathbf{v} \times (\nabla \times \mathbf{E}) + \frac{1}{\sigma}\left(\nabla^2 \mathbf{E} - \frac{\partial^2 \mathbf{E}}{\partial t^2} \right),$$

- Define the magnetic Reynolds number (=convection/diffusion)

$$R_m \equiv LU\sigma$$

where L fm is the characteristic length or time scale, U is the characteristic flow velocity of QGP matter.

Electromagnetic response of the QGP (II)

- If $R_m \ll 1$, one can omit convection terms. The decay is due to diffusion. Diffusion time is $\tau \sim L^2\sigma$: If σ is small (insulator), τ is small, decay fast. If σ is large, τ can be large. Fields may keep constant in the QGP phase.
- If $R_m \gg 1$, one can omit diffusion terms. The magnetic field decay is due to the QGP expansion. In $1+1$ Bjorken picture:

$$\begin{aligned} B_x(t) &= \frac{t_0}{t} B_x^0(t_0), \\ B_y(t) &= \frac{t_0}{t} B_y^0(t_0). \end{aligned}$$

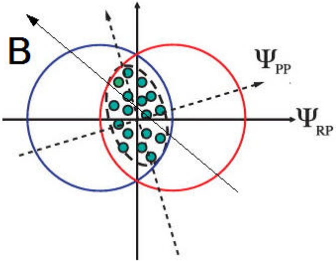
- Theoretical calculation for σ is uncertain: At $T \gtrsim T_c$, $\sigma \approx 6T/e^2$ (**Arnold et al 2003**, $\sigma \approx 7C_{\mathrm{EM}}T$ (**Gupta 2003**), $\sigma \approx 0.4C_{\mathrm{EM}}T$ (**Aarts et al 2007, Ding et al 2010**),$\sigma \approx (1/3)C_{\mathrm{EM}}T$-$C_{\mathrm{EM}}T$ (**Francis:2011**) where $C_{\mathrm{EM}} \equiv \sum_f e_f^2$, $f = u, d, s$, and e_f is quark charge.
- How the electromagnetic field evolves in QGP is sensitive to the electric conductivity.

Histogram of B-angle to matter angle (I)

- E-by-e fluctuations cause field fluctuating azimuthally.
- Matter geometry is characterized by participant nucleons' spatial distribution:

$$\epsilon_1 e^{in\Psi_1} = -\frac{\int d^2\mathbf{r}\rho(\mathbf{r})r^3 e^{i\phi}}{\int d^2\mathbf{r}\rho(\mathbf{r})r^3}$$

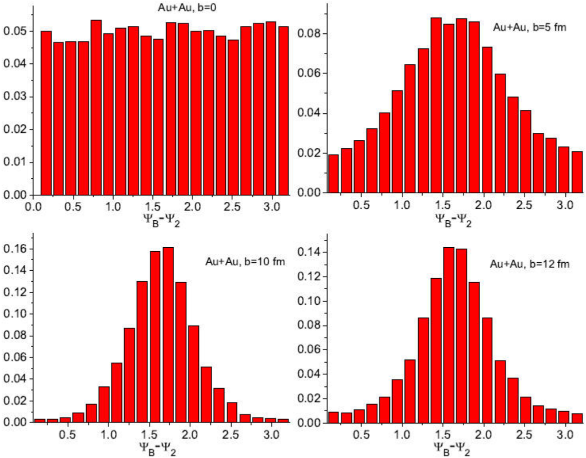
$$\epsilon_n e^{in\Psi_n} = -\frac{\int d^2\mathbf{r}\rho(\mathbf{r})r^n e^{in\phi}}{\int d^2\mathbf{r}\rho(\mathbf{r})r^n}$$



- The second harmonic component Ψ_2 of the participants are particularly important. In figure, $\Psi_{\mathrm{PP}} = \Psi_2$.

Histogram of B-angle to matter angle (II)

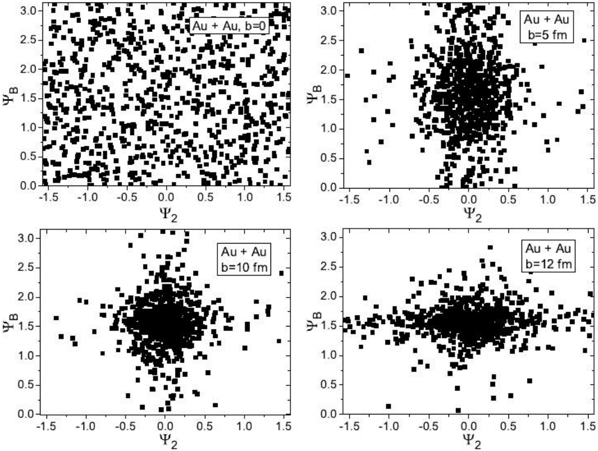
- Histogram of $\Psi_B - \Psi_2$



- For $b \neq 0$, Gaussians peak at $\pi/2$.
- At $b = 0$, no correlation; at $b > 0$, correlation emerge.

Scatter plots

- Scatter plots on Ψ_B-Ψ_2 plane (800 events)



- At $b = 0$, no correlation; at $b > 0$, Ψ_B-Ψ_2 correlation emerge.
- Small b, stronger fluc. along Ψ_B: protons are less than participants; large b, stronger fluc. along Ψ_2: participants are less than protons. Ψ_B and Ψ_2 is strongest correlated around $b = 10$ fm.

Impacts on observables (I)

- Recall that CME-induced dipolar distribution of same-charge:

$$
\begin{aligned}
f_{++} &= A_{++}\cos(\phi_1-\Psi_{\mathbf{B}})\cos(\phi_2-\Psi_{\mathbf{B}}) \\
&\sim \frac{A_{++}}{2}\cos[2(\Psi_{\mathbf{B}}-\Psi_2)]\cos(\phi_1+\phi_2-2\Psi_2)
\end{aligned}
$$

$\Rightarrow$ same-charge correlation $<\cos(\phi_1+\phi_2-2\Psi_2)>=\gamma_{++}\sim\frac{\langle A_{++}\cos[2(\Psi_{\mathbf{B}}-\Psi_2)]\rangle}{2}$. The signal strength $A_{++}\propto\mathbf{B}^2$

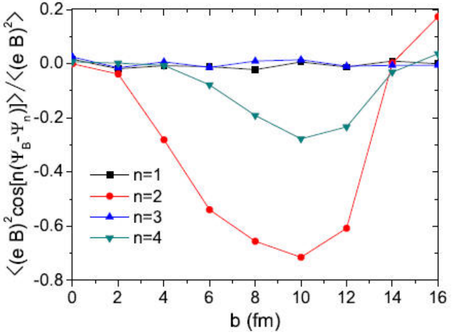
- Similarly, CMW-induced electric quadrupole:

$$
\begin{aligned}
\rho_e(\phi) &\sim 2r_e\cos[2(\phi-\Psi_{\mathbf{B}})] \\
&\sim 2r_e\cos[2(\Psi_{\mathbf{B}}-\Psi_2)]\cos[2(\phi-\Psi_2)]
\end{aligned}
$$

$\Rightarrow v_2(\pi^-)-v_2(\pi^+)=-\langle\frac{r_e}{N_\pm}\cos[2(\Psi_B-\Psi_2)]\rangle$. The signal strength $r_e/N_\pm\propto\mathbf{B}^2$.

Impacts on observables (II)

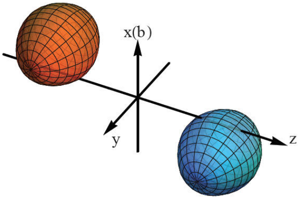
- All osbervables get a suppression factor $R = \langle (e\mathbf{B})^2 \cos[2(\Psi_{\mathbf{B}} - \Psi_2)] \rangle / \langle (e\mathbf{B})^2 \rangle$.
- Correlators $\langle (e\mathbf{B})^2 \cos[n(\Psi_{\mathbf{B}} - \Psi_n)] \rangle / \langle (e\mathbf{B})^2 \rangle$ for $n = 1, 2, 3, 4$.



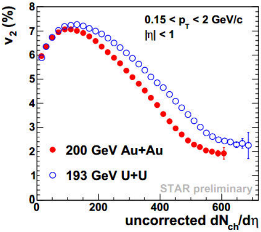
- No correlation between Ψ_B and odd harmonics; For very central and very peripheral events, small correlation between Ψ_B and even harmonics. Coincide with histogram and scatter plots. Strongest correlation happen at $b = 10$ fm, $R \approx -0.7$. $\Rightarrow$ Optimal event class for search of CME, CMW is $b = 10$ fm at RHIC.

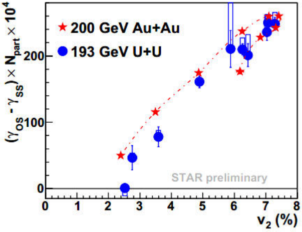
U + U collision

- ^{238}U is highly deformed. Central collision of ^{238}U is promising to disentangle the CME from elliptic flow effect: it is expected no B field but still finite v_2.



- STAR's results (2012) at $\sqrt{s} = 193$ GeV. Support CME.

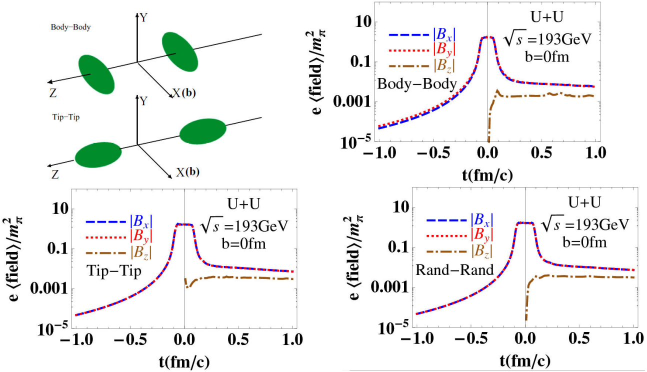




- But fluctuations can lead to large B field?

U + U collision (Preliminary)

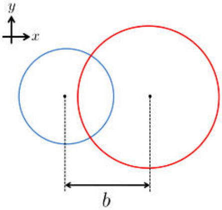
- B-field at central UU collision.

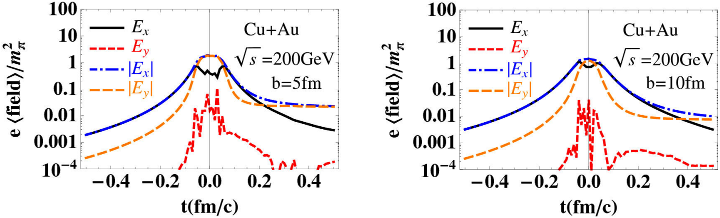


- Although B can be large, no much difference in different orientations.⇒ Indicates the decoupling with matter geometry.⇒ Wouldn't see CME effect.(Need more study: simulation the correlation between B-field orientation and participant plane angle is in progress)

Cu + Au collision (Priliminary)

- Cu + Au collision: geometry induced v_1, v_3 **PHENIX 2012**. We expect strong, in-plane, electric field for noncentral collision.





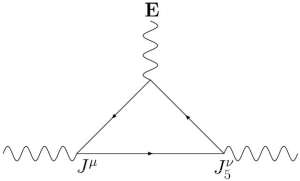
- Indeed, a in-plane, geometry caused, Au to Cu going, E-field! It can cause a finite $v_1(+) - v_1(-)$. (**Hirono, Hongo, and Hirano 2012**).

E-induced anomalous transport?

- Can strong E-field monitor chiral anomaly?
- Generally, in presence of source of chiral anomaly (characterized by μ_5):

$$
\begin{aligned}
j^\mu &= \sigma E^\mu + \lambda B^\mu, \\
j_5^\mu &= \sigma_5 E^\mu + \lambda_5 B^\mu.
\end{aligned}
$$

- Can σ_5 be finite? Consider the triangle anomaly: In presence of $\mathbf{E}$, $\langle J^\mu J_5^\nu \rangle$ could be nonzero.



- But consider chiral fermion, baryon free ($\mu = 0$). E-field only care about charge, not spin $\Rightarrow \sigma_5 = 0$. But if $\mu \neq 0$, the chirality can be attached to charge $\Rightarrow \sigma_5 \neq 0$? Chiral Electric Effect?

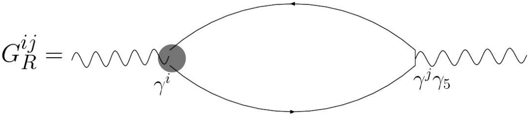
$$\sigma_5 \propto \mu \mu_5.$$

E-induced anomalous transport?

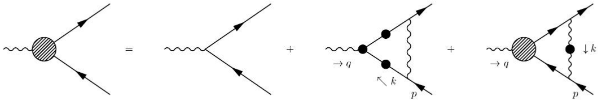
- Apply Kubo formula to hot QED.

$$
\begin{aligned}
\mathbf{j}_5^{\mu}(\omega,\mathbf{k}) &= \sigma_5 i\omega \mathbf{A}(\omega,\mathbf{k}) + \lambda_5 i\mathbf{k}\times\mathbf{A}(\omega,\mathbf{k}) \\
\Rightarrow \ \sigma_5 &= \lim_{\omega\to 0}\lim_{\mathbf{k}\to 0}\frac{i}{3\omega}\sum_{i=1}^{3} G_R^{ii}(\omega,\mathbf{k}) \\
G_R^{ij}(x) &= -i\theta(x)\langle[J_5^i(x), J^j(0)]\rangle.
\end{aligned}
$$

- Leading-log approximation, fermion propagates in axial background so that $\mu_5 \neq 0$



where the effective vertex is (dotted lines: HTL propagators)



It can be verified that Ward identity is satisfied.

E-induced anomalous transport?

- After a long calculation[5], we obtain the leading-log result (Preliminary)

$$\sigma_5 = -\frac{T}{6\pi e^3 \ln(1/e)} \sum_{a,s=\pm} \int_0^\infty dy y^2 \frac{s}{\cosh^2 \frac{y-a(\mu+s\mu_5)/T}{2}} \phi_{as}(y)$$

where $\phi_{as}(y)$ satisfies a differential equation

$$1 = \left[\frac{3}{8y} \coth\frac{y}{2} + \frac{1}{y^2}\right] \phi_{as}(y) + \left[\frac{1}{2} \tanh\frac{y - a(\mu + s\mu - 5)/T}{2} - \frac{1}{y}\right] \times \phi'_{as}(y) - \frac{1}{2} \phi''_{as}(y)$$

- As $\mu, \mu_5 \ll T$:

$$\sigma_5 \propto -\frac{\mu\mu_5}{6\pi T e^3 \ln(1/e)} \propto -\frac{\mu\mu_5}{eT^2}\sigma.$$

Suppressed by $\mu\mu_5/T^2$ but enhanced by $1/e$ comparing to σ.

[5] For σ at $\mu = \mu_5 = 0$: **Basagoiti 2002, Aarts and Resco 2002**

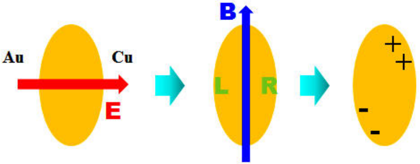
E-induced anomalous transport?

- It can be extended to QCD. The charge current is $\bar{\psi}\gamma^\mu Q\psi$, the anomaly current is $\bar{\psi}\gamma^\mu\gamma_5 A\psi$ which is associated with μ_5, vector current is $\bar{\psi}\gamma^\mu V\psi$ which is associated with μ, where Q, A, V are flavor matrices.

$$\sigma_5 \propto \frac{\mathrm{Tr}(QVA^2)}{\mathrm{Tr}Q^2}\frac{\mu\mu_5}{T^2}\sigma$$

  For $N_f = 2$, V is $U(1)$, A is $U_A(1)$, $\sigma_5 \propto (5/3)(1/e)(\mu\mu_5/T^2)\sigma$

- Possible implication: charge corner-corner correlation in Cu + Au?



- Much smaller (or even opposite sign) amplitudes of $\langle\cos(\phi_i + \phi_j)\rangle$ for both same charges or opposite charges correlations.

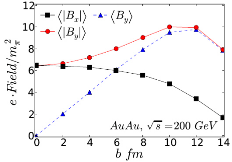
Summary and outlook

- Event-by-event fluctuation can generate very strong magnetic and electric field in HIC.
- The electromagnetic response of QGP is important for EM-field time evolution. Sensitive to σ.
- The e-by-e fluctuation suppress the correlation between $\mathbf{B}$ angle and participant plane angle, but sizable correlation remains for moderate centrality events.
- Optimal centrality class to search CME, CMW is $b = 10$ fm at RHIC AuAu.
- Strong in-plane electric field in Cu + Au collision.
- Possible new anomalous transport induced by electric field.
- Field-matter geometry correlations in U + U and Cu + Au collisions.
- Time evolution of these correlations.
- E-field related anomalous transport. AdS/CFT or kinetic calculations of σ_5.
- ... ...

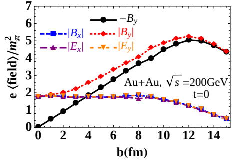
Thank you!

Back up

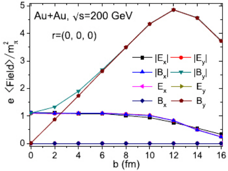
- Different regularization to Lienard-Wiechert potential
  - Short distance cutoff **Bzdak and Skokov 2011**



  - Run a large number of events **Deng and Huang 2012**

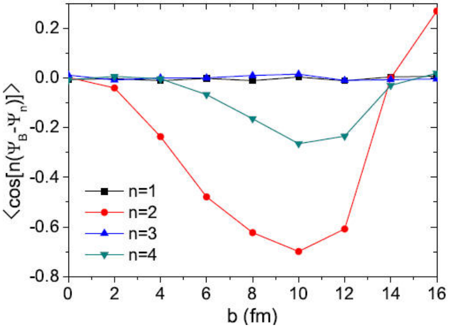


  - Take proton as a uniform charged sphere **Blozynski, Huang, Zhang, and Liao 2012**



Back up

- Correlators $\langle \cos[n(\Psi_{\mathbf{B}} - \Psi_n)] \rangle$ for $n = 1, 2, 3, 4$.



- Very similar with $\langle (e\mathbf{B})^2 \cos[n(\Psi_{\mathbf{B}} - \Psi_n)] \rangle / \langle (e\mathbf{B})^2 \rangle$. $\Rightarrow$ $\mathbf{B}^2$ is not correlated to its orientation.

Back up

- Can strong E-field monitor chiral anomaly?
- Generally, in presence of source of chiral anomaly (characterized by μ_5):

$$\begin{aligned} j^{\mu} &= \sigma\left(E^{\mu}+T\nabla^{\mu}\alpha\right)+\chi T\nabla^{\mu}\alpha_5+\lambda B^{\mu}, \\ j_5^{\mu} &= \sigma_5\left(E^{\mu}+T\nabla^{\mu}\alpha\right)+\chi_5 T\nabla^{\mu}\alpha_5++\lambda_5 B^{\mu}. \end{aligned}$$

The 2nd law implies $\sigma \geq 0$, $\chi_5 \geq 0$, while $\sigma_5+\chi=0$. The terms connecting difference parities do not contribute to entropy production!